# A COMBINED LENGTH-OF-DAY SERIES SPANNING 1832–1997: LUNAR97

by

Richard S. Gross

Space Geodetic Science and Applications Group
Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California

Corresponding Author:

Richard S. Gross
Jet Propulsion Laboratory
Mail Stop 238–332
4800 Oak Grove Drive
Pasadena, CA 91109, USA
ph. +1 818-354-4010
fax +1 818-393-6890
Richard.Gross@jpl.nasa.gov

Friday, August 13, 1999

To be submitted to *Physics of the Earth and Planetary Interiors*

Key words: Earth rotation, Universal time, Length of day, Lunar occultation

# A Combined Length-of-Day Series Spanning 1832–1997: LUNAR97

Richard S. Gross

Jet Propulsion Laboratory, California Institute of Technology, Pasadena.

**Abstract.** Universal time measurements taken by the techniques of lunar occultation, optical astrometry, lunar laser ranging, and very long baseline interferometry have been combined using a Kalman filter to produce a smoothed length-of-day series, LUNAR97, spanning 1832.5–1997.5 at yearly intervals. Decadal length-of-day variations having amplitudes as large as a few milliseconds (ms) are clearly evident in this newly determined series as they have been in previously determined series. Comparing the LUNAR97 length-of-day series with other available series shows differences that are generally less than the uncertainty in the determination of the LUNAR97 length-of-day values, which is about 0.6 ms during the early 19th century, improving by more than an order of magnitude to about 0.03 ms during the late 20th century. Decadal length-of-day variations are therefore a robust feature of the lunar occultation, optical astrometric, and space-geodetic measurements of the Earth's rotation that have been taken since 1832.

## 1. Introduction

Changes in the Earth's rate of rotation are evident by comparing time kept by the rotating Earth, known as universal time, to uniform time scales based either upon atomic clocks, known as atomic time, or upon the motion of the Moon and other celestial bodies, known as ephemeris time. Such comparisons have shown that the Earth's rate of rotation is not constant but exhibits changes on the order of a few parts in $10^8$ with the largest amplitude variations occurring over decades (for recent reviews see Hide and Dickey, 1991; Eubanks, 1993; Rosen, 1993). Investigating the sources and mechanisms of these observed changes in the Earth's rate of rotation requires a series of observations spanning the greatest possible time interval, especially if decadal variations are to be investigated. However, because of the development of new measurement techniques, Earth rotation measurements taken by any single technique are of limited duration. By combining measurements taken by many different techniques, an observed Earth rotation series based upon independent measurements can be generated that spans a greater interval of time than can possibly be spanned by measurements taken by any single technique. The generation of such a long combined series of Earth rotation observations suitable for investigating decadal-scale variations in the Earth's rotation is the subject of this paper.

Universal time has been measured by many different techniques. Prior to the advent of atomic clocks, the most accurate determinations of universal time were obtained by timing the occultations of stars by the Moon (Stephenson, 1997). When atomic clocks became routinely available in 1956 it became possible to determine universal time more accurately by the optical astrometric technique of timing the transit of stars as they pass through the local meridian. The modern space-geodetic techniques of lunar and satellite laser ranging, very long baseline interferometry, and the global positioning system are currently the most accurate techniques available for determining universal time.

Universal time measurements taken by these various techniques have been combined here using a Kalman filter that was developed at the Jet Propulsion Laboratory (JPL) for just this purpose. Kalman filters are commonly used for parameter estimation when stochastic models describing the system are available and when the measurements contain noise (e.g., Nahi, 1969; Gelb, 1974; Bierman, 1977). Since Kalman filters can also employ stochastic models for the growth in the uncertainty between measurements, realistic errors of the parameters being estimated by the Kalman filter can also be obtained. A Kalman filter for combining and predicting Earth orientation parameters was developed at JPL to support the tracking and navigation of interplanetary spacecraft (Morabito et al., 1988). Freedman et al. (1994) and Hamdan and Sung (1996) describe the models of universal time and length-of-day employed by JPL's Kalman Earth orientation filter and Gross et al. (1998) discuss the use of it to combine Earth orientation measurements. Here it is used to combine lunar occultation, optical astrometric, and space-geodetic measurements of universal time and to determine the corresponding length-of-day (LOD) values and uncertainties.

Universal time (UT) is a measure of the angle through which the Earth has rotated at any given instant. The Earth's angular velocity is just the time rate-of-change of this rotation angle (that is, of universal time), and the length-of-day $\Lambda(t)$ is just the period of the Earth's rotation. Thus, excess length-of-day variations $\Delta\Lambda(t)$ are related to the difference between universal time UT1 and atomic time TAI by (e.g., Lambeck, 1980, p. 63):

$$\Delta\Lambda(t) = -\Lambda_o \frac{d\,(\mathrm{UT1{-}TAI})}{dt} \tag{1}$$

where $\Lambda_o$ is the nominal length of the day (86400 seconds). Henceforth in this paper, LOD shall be taken to mean the excess $\Delta\Lambda(t)$ of the length of the day over its nominal length of 86400 seconds rather than its instantaneous value $\Lambda(t)$.

## 2. Earth rotation measurements combined

LUNAR97 is based upon universal time measurements taken by the techniques of lunar occultation, optical astrometry, lunar laser ranging, and very long baseline interferometry. The lunar occultation series used here is that of Jordi et al. (1994) which spans 1830.0–1955.5 at 4-month intervals. This series is extended to the present using COMB97 (Gross, 1999), a recent combination of optical astrometric and space-geodetic measurements spanning 1962.0 to 1998.0 at 5-day intervals. The gap between the end of the lunar occultation series (1955.5) and the start of COMB97 (1962.0) is bridged by using the recently available Hipparcos optical astrometric series (Vondrák et al., 1998) whose UT1 component spans 1956.0 to 1992.0 at 5-day intervals. Prior to combining these series together and determining the length-of-day values, they must first be made compatible with each other. Since COMB97 is already available, this is accomplished by making the lunar occultation and Hipparcos optical astrometric series compatible with COMB97.

## 2.1. COMB97

COMB97 is a combination of Earth orientation measurements (universal time, polar motion, and linear combinations thereof) taken by the techniques of optical astrometry, lunar and satellite laser ranging, very long baseline interferometry (VLBI), and the global positioning system (Gross, 1999). Prior to their combination, each independent series of measured Earth orientation values was preprocessed in order to: (1) remove leap seconds and both solid Earth and ocean tidal terms from the universal time measurements, (2) adjust the stated uncertainties of the measurements so that the residual of each series had a reduced chi-square of one when compared to a combination of all other independent series, (3) delete those outlying data points whose residual values were greater than three times their adjusted uncertainties, and (4) adjust the bias and rate of each series so that they were all in alignment with each other and were consistent with the International Terrestrial Reference Frame ITRF96 (Boucher et al., 1998). In addition, the annual term of the Bureau International de l'Heure (BIH) optical astrometric series was adjusted so that it agreed with the annual term exhibited by a combination of the space-geodetic series. This was done in order to correct for known systematic errors in optical astrometric measurements at the annual frequency. The adjusted series were then combined together using JPL's Kalman Earth orientation filter, after which leap seconds and both solid Earth and ocean tidal terms were restored to the UT1 and LOD components. The resulting combined Earth orientation series, COMB97, spans 1962.0–1998.0 at 5-day intervals and consists of values and $1\sigma$ standard errors for UT1–UTC, the $x$- and $y$-components of polar motion, and their rates of change, including LOD.

The UT1 and LOD components of COMB97 are based upon optical astrometric measurements of universal time spanning 1962.0–1982.0, lunar laser ranging measurements spanning 1970.3–1996.8, and VLBI measurements spanning 1978.8–1998.0 (Gross, 1999). For LUNAR97, the LOD component of COMB97 was used after first removing the solid Earth and ocean tidal terms from the LOD values. The model of Yoder et al. (1981) was used to remove the effect of the solid Earth tides at all tidal periods between 5 days and 18.6 years, and the model of

Desai (1996) was used to remove the dynamic (non-equilibrium) effect of the ocean tides at the fortnightly and monthly tidal periods.

### *2.2. Hipparcos*

The Hipparcos astrometric satellite was launched in 1989 in order to accurately measure the positions, proper motions, and parallaxes of about 100,000 stars. The resulting final Hipparcos star catalog has been recently used to re-reduce past optical astrometric measurements of latitude and longitude in order to derive a homogenous series of universal time, polar motion, and nutation values (Vondrák, 1991; Vondrák et al., 1992, 1995, 1997, 1998). The extant optical astrometric measurements, numbering 4,315,628 taken at 48 instruments during 1899.7–1992.0, were collected by Vondrák and his colleagues and were corrected by them for instrumental effects and such systematic effects as plate tectonic motion, ocean loading, and tidal variations. Using the final Hipparcos star catalog, the corrected measurements were then used by them to solve for Earth orientation parameters using current astronomical standards and data reduction techniques. The resulting Earth orientation series, which will be referred to here as the Hipparcos series, consists of values and uncertainties for polar motion and nutation spanning 1899.7 to 1992.0 at unequal but nearly 5-day intervals, and for UT1–TAI spanning 1956.0 to 1992.0 at the same unequal but nearly 5-day intervals.

Prior to using the Hipparcos UT1–TAI values to determine the excess length-of-day for LUNAR97, they were first pre-processed to make them compatible with COMB97 by: (1) correcting them to be consistent with the new definition of Greenwich Sidereal Time (GST) as adopted by the International Earth Rotation Service (IERS; IERS, 1997, p. I49), and (2) removing solid Earth and ocean tidal terms. In addition, the stated uncertainties of the Hipparcos UT1–TAI values were adjusted to make them consistent with the scatter evident in its residual series, and outlying data points were deleted (see below). Consistency with the new definition of GST was imposed by adding to the Hipparcos UT1–TAI values the periodic terms:

$$\Delta(\mathrm{UT1-TAI}) = -0.176 \sin(\Omega) - 0.004 \sin(2\Omega) \tag{2}$$

where $\Omega$ is the mean longitude of the lunar ascending node which varies with a period of about 18.6 years, and the amplitude of the periodic terms are given in milliseconds (ms). The effect of these periodic terms on the length-of-day is very small, with the amplitude of the 18.6-year and 9.3-year terms being only 0.163 microseconds ($\mu$s) and 0.008 $\mu$s, respectively.

Tidal terms were removed from the Hipparcos UT1–TAI values using the same models that were used to remove them from COMB97. That is, the model of Yoder et al. (1981) was used to remove the effect of the solid Earth tides at all tidal periods between 5 days and 18.6 years, and the model of Desai (1996) was used to remove the dynamic (non-equilibrium) effect of the ocean tides at the fortnightly and monthly tidal periods. As was done for the BIH series in COMB97 (Gross, 1999), the amplitude of the ocean (but not solid Earth—see below) tidal terms of monthly and shorter period were attenuated prior to their removal in order to account for the reduction in amplitude of these terms caused by the nearly 5-day averaging interval of the Hipparcos values. The amplitude attenuation factor that was applied is a function of the tidal period and averaging interval and is given by Gross et al. (1998, pp. 226–227). However, unlike the BIH series in COMB97, the amplitude of the solid Earth tidal terms in the Hipparcos series were not attenuated prior to their removal because the solid Earth (but not ocean) tidal terms of period less than 35 days were removed from the raw optical astrometric observables when reducing them to determine the Earth orientation parameters (Vondrák et al., 1998, sec. 8.2). Since the full amplitude of the solid Earth tidal terms were subsequently restored to the universal time component to obtain the released Hipparcos UT1–TAI values, the full amplitude of the solid Earth tidal terms should now be removed.

The stated uncertainties of the Hipparcos UT1–TAI values were adjusted in order to make them consistent with the scatter evident in the residual series formed by differencing the raw with the smoothed Hipparcos UT1–TAI values, where the smoothed values were obtained by Kalman filtering the raw values using JPL's Kalman Earth orientation filter. In particular, the uncertainties of the Hipparcos UT1–TAI values were adjusted by applying to them a multiplicative scale factor that made the raw minus smoothed residual series have a reduced chi-square of one. An iterative procedure was required to accomplish this since the degree of smoothing applied by the Kalman filter is a function of the uncertainties that are being adjusted. In order to minimize interpolation error (see Gross et al., 1998, pp. 223–225), the comparison of the raw Hipparcos values with the smoothed values was done at the epochs of the measurements by using the Kalman filter to interpolate to, and print its estimates at, the exact epochs of those measurements. In addition, during this iterative uncertainty adjustment procedure, data points whose residual values were greater than three times their adjusted uncertainties were considered to be outliers and were therefore deleted. A total of 35 of the 2630 Hipparcos UT1–TAI values spanning 1956.0–1992.0 were thus deleted, with 3 of the deleted values occurring during 1956.0–1962.0.

After the Hipparcos UT1–TAI values were pre-processed as described above, they: (1) are consistent with the new definition of GST, as are the COMB97 universal time values (Gross, 1999), (2) have had solid Earth and ocean tidal terms removed, (3) have had their uncertainties inflated by a factor of 2.12, and (4) have had outlying data points deleted. The pre-processed Hipparcos universal time series was then Kalman-filtered using JPL's Kalman Earth orientation filter in order to obtain the excess length-of-day values to be used in LUNAR97. When doing this, the Kalman filter printed its estimates at the same equally spaced 5-day intervals at which the COMB97 values are given so that the resulting Hipparcos and COMB97 excess length-of-day series could simply be concatenated for LUNAR97 (see Section 3.1 below).

The procedure used to adjust the stated uncertainties of the Hipparcos UT1–TAI values seems reasonable since the uncertainties of the resulting Hipparcos length-of-day values are quite close to those of COMB97 near 1962.0 (see Figure 1b). During 1962.0–1970.3 the COMB97 universal time and length-of-day values are based solely upon the BIH optical astrometric measurements (Gross, 1999). For COMB97, the stated uncertainties of the BIH universal time values were adjusted so that its residual series had a reduced chi-square of one, where the residual series was formed by differencing the BIH series with a combination of more accurate space-geodetic measurements. Since the adjusted BIH uncertainties are consistent with the scatter in this residual series, and are hence reasonably realistic, then the uncertainties of the COMB97 length-of-day values during 1962.0–1970.3 should also be reasonably realistic since they are based solely upon the BIH UT1–TAI values and were determined using a Kalman filter that included a stochastic model for the growth in the uncertainty between measurements. Furthermore, since both the Hipparcos and COMB97 series during 1962.0–1970.3 are based solely upon measurements taken by the same optical astrometric technique, and since the procedure used to adjust the stated uncertainties of the Hipparcos UT1–TAI values yields length-of-day uncertainties that are equivalent to those of COMB97 near 1962.0, then the Hipparcos LOD uncertainties should also be reasonably realistic. Because the procedure used to adjust the uncertainties of the Hipparcos UT1–TAI values has apparently yielded reasonably realistic uncertainties for the Hipparcos length-of-day values, it will also be used below to adjust the stated uncertainties of the lunar occultation series.

### *2.3. Lunar occultation*

The most recent re-reduction of the lunar occultation measurements for universal time is that of Jordi et al. (1994) who analyzed about 53,000 observations of lunar occultations that spanned 1830.0–1955.5. Compared to previous re-reductions of the lunar occultation measurements, they derived an improved universal time series by using an improved lunar ephemeris (LE200) and an improved reference frame, namely, that defined by the FK5 star catalog. The universal time series they obtained consists of values and $1\sigma$ standard errors for the difference between terrestrial time and universal time (TT–UT1) spanning 1830.0–1955.5 at 4-month intervals. Terrestrial time (TT) is an extension of ephemeris time and, to sufficient accuracy for the present purpose, is related to international atomic time TAI by (e.g., Seidelmann, 1992, chap. 2):

$$\mathrm{TT} = \mathrm{TAI} + 32.184 \text{ seconds.} \qquad (3)$$

The distribution of lunar occultation measurements within a synodic month, whose duration is about 29.5306 mean solar days, is not uniform but peaks about 9 days after the start of the month because most of the measurements are taken when the star is occulted by the dark, leading edge of the Moon (Jordi et al., 1994, Fig. 3). Because of this non-uniform distribution of occultation measurements within a synodic month, the mean epoch assigned to the derived universal time value should not be the mid-point of the month, but should rather be 9 days after the start of the month. Here, since the Jordi et al. (1994) universal time values are average values based upon lunar occultation measurements taken during 4 consecutive months, and since within each month the distribution of occultation measurements peaks 9 days after the start of the month, the mean epoch assigned to the Jordi et al. (1994) universal time value was 53.296 days after the start of the 4-month-long observing interval. This mean epoch was determined by simply averaging the 4 hypothetical epochs that would have been assigned to each of the 4 hypothetical monthly universal time values that could have been derived from the lunar occultation measurements taken during each synodic month of the 4-month-long observing interval.

Prior to using the TT–UT1 values of Jordi et al. (1994) to determine the excess length-of-day for LUNAR97 they were first pre-processed to make them compatible with COMB97 and the Hipparcos series by: (1) using Eq. (3) to convert them to values of UT1–TAI, (2) using Eq. (2) to make them consistent with the new definition of Greenwich Sidereal Time, (3) adjusting them to be consistent with the most recent value for the tidal acceleration of the Moon (see below), and (4) removing solid Earth and ocean tidal terms. In addition, the stated uncertainties of the Jordi et al. (1994) universal time values were adjusted to make them consistent with the scatter evident in its residual series, and outlying data points were deleted (see below).

The most recent estimate for the observed tidal acceleration of the Moon is −25.8 arcseconds/century$^2$ ("/cy$^2$; Williams, personal communication, 1998). However, the value for the tidal acceleration of the Moon implicit in the LE200 lunar ephemeris is −23.9"/cy$^2$ (Jordi et al., 1994, Sec. 5). The $\Delta T \equiv$ TT–UT1 values of Jordi et al. (1994) were therefore adjusted here to make them consistent with the most recent estimate for the observed tidal acceleration of the Moon $\dot{n}_{obs}$ by applying the correction (e.g., Lieske, 1987, Eq. 3):

$$\Delta T(\dot{n}_{obs}) \;=\; \Delta T(\dot{n}_{LE200}) \;-\; \frac{1}{2}\,1.82144\,(\dot{n}_{obs} - \dot{n}_{LE200})\,T_o^2 \qquad (4)$$

where $\Delta T(\dot{n}_{obs})$ is the corrected value of TT–UT1, $\Delta T(\dot{n}_{LE200})$ is the value of TT–UT1 derived by Jordi et al. (1994) using the tidal acceleration of the Moon $\dot{n}_{LE200}$ implicit in the LE200 lunar ephemeris, the reciprocal of the mean motion of the Moon is 1.82144 seconds/arcsecond, and $T_o$ is the epoch of the $\Delta T$ observation in Julian centuries since 1955.5. From Eq. (1), this correction to $\Delta T$ arising from a difference of $-1.9''/\mathrm{cy}^2$ in the tidal acceleration of the Moon corresponds to a correction to the secular change in the length of the day of only +0.095 ms/cy.

Tidal terms were removed from the Jordi et al. (1994) universal time values using the same models that were used to remove them from both COMB97 and the Hipparcos series. However, since the Jordi et al. (1994) universal time values are averages over 4 months, only those tidal terms having periods greater than 4 months were removed. That is, the model of Yoder et al. (1981) was used to remove the effect of the solid Earth tides at all tidal periods between 118 days and 18.6 years. No ocean tidal terms were removed since the longest period term in the Desai (1996) model has a period of 27.56 days, and so these ocean tidal terms should have already been removed from the 4-month-averaged Jordi et al. (1994) universal time values. As was done with the Hipparcos series and the BIH series in COMB97, the amplitude of the solid Earth tidal terms were attenuated prior to their removal in order to account for the reduction in amplitude of these terms caused by the 4-month-long averaging interval of the Jordi et al. (1994) universal time values. The amplitude attenuation factor that was applied is a function of the tidal period and averaging interval and is given by Gross et al. (1998, pp. 226–227).

As was also done with the Hipparcos optical astrometric series, the stated uncertainties of the Jordi et al. (1994) universal time values were adjusted in order to make them consistent with the scatter evident in the residual series formed by differencing the raw with the smoothed values, where the smoothed values were obtained by Kalman filtering the raw values using JPL's Kalman Earth orientation filter. In particular, the uncertainties of the Jordi et al. (1994) universal time values were adjusted by applying to them a multiplicative scale factor that made the raw minus smoothed residual series have a reduced chi-square of one. As with the Hipparcos series, an iterative procedure was required to accomplish this since the degree of smoothing applied by the Kalman filter is a function of the uncertainties being adjusted. In order to minimize interpolation error (see Gross et al., 1998, pp. 223–225), the comparison of the raw Jordi et al. (1994) values with the smoothed values was done at the assigned epochs of the measurements by using the Kalman filter to interpolate to, and print its estimates at, the exact epochs of those measurements. In addition, during this iterative uncertainty adjustment procedure, data points whose residual values were greater than three times their adjusted uncertainties were considered to be outliers and were therefore deleted. A total of 44 of the 386 Jordi et al. (1994) universal time values spanning 1830.0–1955.5 were thus deleted.

After the Jordi et al. (1994) universal time values were pre-processed as described above, they: (1) have been converted to values of UT1–TAI, (2) are consistent with the new definition of GST, (3) are consistent with the most recent estimate for the tidal acceleration of the Moon, (4) have had solid Earth and ocean tidal terms removed, (5) have had their uncertainties adjusted by a factor of 0.977, and (6) have had outlying data points deleted. The pre-processed Jordi et al. (1994) universal time series was then Kalman-filtered using JPL's Kalman Earth orientation filter in order to obtain the excess length-of-day values to be used in LUNAR97. When doing this, the Kalman filter printed its LOD estimates at the same epochs as the input universal time values.

## 3. LUNAR97

By pre-processing the lunar occultation, Hipparcos, and COMB97 series as described above in Section 2, they have been made compatible with each other by: (1) correcting them to agree with the new definition of GST, (2) removing the same solid Earth and ocean tidal terms from them in the same manner using the same tide models for these terms, (3) adjusting their uncertainties in the same manner by applying a multiplicative scale factor that makes the respective residual series have a reduced chi-square of one, and (4) deleting those outlying data points whose respective residual values were greater than three times their adjusted uncertainties. LUNAR97 is now ready to be created by merging the resulting length-of-day series, smoothing and interpolating the result so that its values are given at equally spaced intervals of 1 year.

### *3.1. Merging and smoothing the Hipparcos and COMB97 series*

Without further adjustment, the pre-processed Hipparcos length-of-day series spanning 1956.0–1962.0 was simply concatenated with the pre-processed COMB97 length-of-day series spanning 1962.0–1998.0. No adjustment to align these series with each other prior to concatenation was deemed necessary since their overlap near 1962.0 showed no significant bias. The merged Hipparcos and COMB97 LOD series, spanning 1956.0–1998.0 at 5-day intervals, was then smoothed in order to remove seasonal and shorter period variations. This was accomplished by taking a simple average of the length-of-day values within a running, non-overlapping window of length 1 year. Since the sample autocorrelation function of the merged Hipparcos and COMB97 LOD series indicated an effective decorrelation time greater than 1 year, the values being averaged within each window of length 1 year are not independent. The uncertainties assigned to the averaged values were therefore determined by simply averaging the variances in the same manner that the values themselves were averaged, taking the square root of the result. The final merged and smoothed Hipparcos and COMB97 LOD series spans 1956.5–1997.5 at unequal but nearly 1-year intervals.

### *3.2. Smoothing the lunar occultation series*

The pre-processed Jordi et al. (1994) lunar occultation length-of-day series was first smoothed before merging it with the concatenated Hipparcos and COMB97 series. The degree of smoothing applied was chosen so that the variability of the smoothed lunar occultation LOD series matched that of the merged and smoothed Hipparcos and COMB97 series. In order to accomplish this, the lunar occultation values during 1832.0–1880.0 were smoothed over 5-year intervals, with the values during 1880.0–1955.5 being smoothed over 3-year intervals. A higher degree of smoothing needed to be applied to the pre-1880 values because of the greater amount of noise present before 1880. This increased noise is due to the approximately 4-fold decrease in the number of lunar occultation observations taken before about 1880 (Jordi et al., 1994, Fig. 1). The increased noise before 1880 was also evident in the residual lunar occultation universal time series (not shown) that was generated during the uncertainty adjustment procedure described above in Section 2.3. As with the Hipparcos and COMB97 series, the smoothing was applied to the LOD values by taking a simple average of them within a running window of length 3 or 5 years, with successive windows being offset by 1 year in order to form smoothed values at 1-year intervals. Successive smoothed lunar occultation LOD values are therefore not independent.

In order to avoid introducing a gap in the smoothed series at 1955.5, the 3-year-averaged series was actually generated by smoothing the merged lunar occultation and Hipparcos series. The pre-processed Jordi et al. (1994) lunar occultation series was first concatenated with the pre-processed Hipparcos series after it had been smoothed by taking a simple average of its values within a running, non-overlapping window of length 4 months. The concatenated 4-month-averaged lunar occultation and Hipparcos series was then smoothed over 3-year intervals as described above in order to obtain the 3-year-averaged lunar occultation values to be used for LUNAR97.

Since the sample autocorrelation function of the pre-processed Jordi et al. (1994) lunar occultation series indicated an effective decorrelation time greater than 5 years, the values being averaged within each window, regardless of whether its length is 3 or 5 years, are not independent. Thus, as with the smoothed Hipparcos and COMB97 LOD series, the uncertainties assigned to the averaged lunar occultation values were determined by simply averaging the variances in the same manner that the values themselves were averaged, taking the square root of the result. The final 5-year-smoothed lunar occultation LOD series spans 1832.5–1879.5 at unequal but nearly 1-year intervals, and the final 3-year-smoothed lunar occultation LOD series spans 1880.5–1955.5 at unequal but nearly 1-year intervals.

### *3.3. Final LUNAR97 series*

Without further adjustment, the 5-year-smoothed lunar occultation LOD values spanning 1832.5–1879.5 were simply concatenated with the 3-year-smoothed lunar occultation LOD values spanning 1880.5–1955.5 which were in turn simply concatenated with the merged 1-year-smoothed Hipparcos and COMB97 LOD values spanning 1956.5–1997.5. No adjustment to align these series with each other prior to concatenation was deemed necessary since no significant bias between the series was evident. Values at equally spaced intervals of 1 year were then obtained by linear interpolation. The final LUNAR97 excess length-of-day series, which is given in Table 1 and displayed in Figure 1, consists of values and uncertainties for the length-of-day spanning 1832.5–1997.5 at 1-year intervals.

## 4. Discussion

In Figure 2, the LUNAR97 length-of-day series (black curve with gray shading depicting the $\pm\, 1\sigma$ uncertainty of the values) is compared with both the McCarthy and Babcock (1986) LOD series (red curve) as well as with that derived by Stephenson and Morrison (1984; green curve). The McCarthy and Babcock (1986) LOD series is based upon the lunar occultation universal time series of Martin (1969; spanning 1627–1860) and Morrison (1979; spanning 1860–1943), both of whom used the Improved Lunar Ephemeris ($j$ = 2) and a reference frame defined by the FK4 star catalog when reducing the lunar occultation measurements for universal time. During 1943–1955 the McCarthy and Babcock (1986) LOD series is based upon universal time values derived from lunar occultation measurements provided to them by L. Morrison, during 1956–1962 it is based upon universal time measurements made using the photographic zenith tubes at the United States Naval Observatory, and during 1962–1984 it is based upon universal time values obtained from the BIH. The Stephenson and Morrison (1984) LOD series is based upon their own reduction of the lunar occultation measurements published by Morrison et al. (1981; spanning 1623–1942) and Morrison (1978; spanning 1943–1971) as well as upon timings of lunar and solar eclipses. The $j$ = 2 lunar ephemeris and the reference frame defined by the FK4 star catalog were also used by Stephenson and Morrison (1984) when reducing the lunar occultation measurements for universal time and LOD. As can be seen, the differences between the LUNAR97, McCarthy and Babcock (1986), and Stephenson and Morrison (1984) LOD series are generally less than the uncertainty of the LUNAR97 LOD values. In particular, the decadal variations exhibited by these 3 different LOD series are very similar, with the differences between the series occurring primarily at higher frequencies. Thus, differences in lunar ephemeris, reference frame, and analysis technique have not changed the nature of the observed decadal variations in the length-of-day derived from lunar occultation measurements taken since 1830, optical astrometric measurements taken since 1956, and space-geodetic measurements taken since 1970. The decadal LOD variations therefore appear to be a robust feature of the lunar occultation, optical astrometric, and space-geodetic measurements.

## 5. Summary

A smoothed series of excess length-of-day values and their uncertainties suitable for investigating decadal-scale variations has been derived by using JPL's Kalman Earth orientation filter to combine and differentiate lunar occultation measurements of universal time spanning 1830.0–1955.5, Hipparcos optical astrometric measurements of universal time spanning 1956.0–1962.0, BIH optical astrometric measurements of universal time spanning 1962.0–1982.0, lunar laser ranging measurements of universal time spanning 1970.3–1996.8, and very long baseline interferometry measurements of universal time spanning 1978.8–1998.0. Solid Earth and ocean tidal terms have been removed from the derived length-of-day values and the lunar occultation measurements were adjusted to be consistent with the most recent estimate of the tidal acceleration of the Moon. In order to produce a homogeneous LOD series, and accounting for differences in the noise content of the various measurements, the length-of-day values were smoothed over 5-year intervals during 1832.0–1880.0, 3-year intervals during 1880.0–1955.5, and 1-year intervals during 1956.0–1998.0. The final series, LUNAR97, spans 1832.5–1997.5 at 1-year intervals and consists of values and $1\sigma$ standard errors for the excess length-of-day.

When deriving the LUNAR97 length-of-day series, particular attention was given to obtaining reasonably realistic estimates of the uncertainties assigned to the derived LOD values. The stated uncertainties of each series of universal time measurements were adjusted so that they were consistent with the scatter evident in their respective residual series, thereby assuring that the uncertainties assigned to the universal time values going into the Kalman Earth orientation filter are reasonably realistic. Furthermore, the Kalman Earth orientation filter used to combine the universal time measurements and determine the excess length-of day values and uncertainties contains stochastic models for both the universal time / length-of-day process as well as for the growth in the uncertainty between the universal time measurements. Since the uncertainties of the derived length-of-day values are based upon the input adjusted universal time uncertainties as well as upon the stochastic models contained within the Kalman filter, and since the input universal time uncertainties and stochastic models are reasonably realistic, then the uncertainties assigned by the Kalman filter to the derived excess length-of-day values should also be reasonably realistic.

**Acknowledgments.** The work described in this paper was performed at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration. Funding for this work was provided by the Geodynamics and Geopotential Fields program of NASA's Office of Earth Science.

## References

Bierman, G.J., 1977. Factorization Methods for Discrete Sequential Estimation. Academic Press, New York.

Boucher, C., Altamimi, Z. and Sillard, P., 1998. Results and analysis of the ITRF96, Int. Earth Rot. Serv. Tech. Note 24, 172 pp., Obs. de Paris, Paris.

Desai, S.D., 1996. Ocean tides from Topex/Poseidon altimetry with some geophysical applications. Ph.D. Thesis, 254 pp., Univ. of Colo., Boulder.

Eubanks, T.M., 1993. Variations in the orientation of the Earth. In: D.E. Smith and D.L. Turcotte (Editors), Contributions of Space Geodesy to Geodynamics: Earth Dynamics. American Geophysical Union Geodynamics Series Vol. 24, Washington, DC, pp. 1–54.

Freedman, A.P., Steppe, J.A., Dickey, J.O., Eubanks, T.M. and Sung, L.-Y., 1994. The short-term prediction of universal time and length of day using atmospheric angular momentum. J. Geophys. Res., 99:6981–6996.

Gelb, A. (Editor), 1974. Applied Optimal Estimation. MIT Press, Cambridge, Mass.

Gross, R.S., 1999. Combinations of Earth orientation measurements: SPACE97, COMB97, and POLE97. J. Geodesy, in press.

Gross, R.S., Eubanks, T.M., Steppe, J.A., Freedman, A.P., Dickey, J.O. and Runge, T.F., 1998. A Kalman-filter-based approach to combining independent Earth-orientation series. J. Geodesy, 72: 215–235.

Hamdan, K. and Sung, L.-Y., 1996. Stochastic modeling of length of day and universal time, J. Geodesy, 70:307–320.

Hide, R. and Dickey, J.O., 1991. Earth's variable rotation, Science, 253:629–637.

IERS, 1997. 1996 IERS Annual Report, Obs. de Paris, Paris.

Jordi, C., Morrison, L.V., Rosen, R.D., Salstein, D.A. and Rosselló, G., 1994. Fluctuations in the Earth's rotation since 1830 from high-resolution astronomical data. Geophys. J. Int., 117: 811–818.

Lambeck, K., 1980. The Earth's Variable Rotation: Geophysical Causes and Consequences. Cambridge University Press, New York.

Lieske, J.H., 1987. Galilean satellite evolution: Observational evidence for secular changes in mean motions. Astron. Astrophys., 176:146–158.

Martin, C.F., 1969. A study of the rate of rotation of the Earth from occultations of stars by the Moon 1627–1860. Ph.D. Thesis, Yale University, New Haven.

McCarthy, D.D. and Babcock, A.K., 1986. The length of day since 1656. Phys. Earth Planet. Int., 44:281–292.

Morabito, D.D., Eubanks, T.M. and Steppe, J.A., 1988. Kalman filtering of Earth orientation changes. In: A.K. Babcock and G.A. Wilkins (Editors),The Earth's Rotation and Reference Frames for Geodesy and Geodynamics. D. Reidel, Dordrecht, Holland, pp. 257–267.

Morrison, L.V., 1978. Catalogue of observations of occultations of stars by the Moon for the years 1943 to 1971. Roy. Greenwich Obs. Bull., No. 183.

Morrison, L.V., 1979. Re-determination of the decade fluctuations in the rotation of the Earth in the period 1861–1978. Geophys. J. Roy. astr. Soc., 58:349–360.

Morrison, L.V., Lukac, M.R. and Stephenson, F.R., 1981. Catalogue of observations of occultations of stars by the Moon for the years 1623–1942 and solar eclipses for the years 1621–1806. Roy. Greenwich Obs. Bull., No. 186.

Nahi, N.E., 1969. Estimation Theory and Applications. John Wiley and Sons, New York.

Rosen, R.D., 1993. The axial momentum balance of Earth and its fluid envelope. Surveys Geophys., 14:1–29.

Seidelmann, P.K. (Editor), 1992. Explanatory Supplement to the Astronomical Almanac. University Science Books, Mill Valley, Calif.

Stephenson, F.R., 1997. Historical Eclipses and Earth's Rotation. Cambridge University Press, New York.

Stephenson, F.R. and Morrison, L.V., 1984. Long-term changes in the rotation of the Earth: 700 B.C. to A.D. 1980. Phil. Trans. Roy. Soc. London, A313:47–70.

Vondrák, J., 1991. Calculation of the new series of the Earth orientation parameters in the HIPPARCOS reference frame. Bull. Astron. Inst. Czechosl., 42: 283–294.

Vondrák, J., Feissel, M. and Essaïfi, N., 1992. Expected accuracy of the 1900–1990 Earth orientation parameters in the Hipparcos reference frame. Astron. Astrophys., 262: 329–340.

Vondrák, J., Ron, C., Pesek, I. and Cepek, A., 1995. New global solution of Earth orientation parameters from optical astrometry in 1900–1990. Astron. Astrophys., 297: 899–906.

Vondrák, J., Ron, C. and Pesek, I., 1997. Earth rotation in the Hipparcos reference frame. Celes. Mech. Dyn. Astron., 66: 115–122.

Vondrák, J., Pesek, I., Ron, C. and Cepek, A., 1998. Earth orientation parameters 1899.7–1992.0 in the ICRS based on the Hipparcos reference frame. Publication No. 87 of the Astronomical Institute of the Academy of Sciences of the Czech Republic, 56 pp., Ondrejov, Czech Republic.

Yoder, C.F., Williams, J.G. and Parke, M.E., 1981. Tidal variations of Earth rotation. J. Geophys. Res., 86: 881–891.

**Table 1.** LUNAR97 Combined Length-of-Day Series

| Date (yr) | LOD (ms) | Error (ms) | Date (yr) | LOD (ms) | Error (ms) | Date (yr) | LOD (ms) | Error (ms) | Date (yr) | LOD (ms) | Error (ms) | Date (yr) | LOD (ms) | Error (ms) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1832.5 | -1.3254 | 0.6011 | 1866.5 | -2.4450 | 0.4739 | 1900.5 | 3.3759 | 0.4041 | 1934.5 | -0.0909 | 0.3433 | 1968.5 | 2.6225 | 0.1004 |
| 1833.5 | -1.0422 | 0.5705 | 1867.5 | -2.8397 | 0.4908 | 1901.5 | 3.5997 | 0.4070 | 1935.5 | -0.1788 | 0.3418 | 1969.5 | 2.8259 | 0.0981 |
| 1834.5 | -0.6979 | 0.5670 | 1868.5 | -3.2369 | 0.5040 | 1902.5 | 3.6805 | 0.4128 | 1936.5 | -0.0946 | 0.3487 | 1970.5 | 2.8452 | 0.0954 |
| 1835.5 | -0.4074 | 0.5600 | 1869.5 | -3.2728 | 0.5176 | 1903.5 | 3.4660 | 0.4165 | 1937.5 | 0.1851 | 0.3507 | 1971.5 | 3.0067 | 0.0890 |
| 1836.5 | -0.1781 | 0.5495 | 1870.5 | -2.9486 | 0.5268 | 1904.5 | 3.1690 | 0.4187 | 1938.5 | 0.5489 | 0.3573 | 1972.5 | 3.1920 | 0.0810 |
| 1837.5 | 0.0231 | 0.5354 | 1871.5 | -2.6015 | 0.5361 | 1905.5 | 3.1710 | 0.4215 | 1939.5 | 0.6794 | 0.3685 | 1973.5 | 3.0683 | 0.0804 |
| 1838.5 | 0.0923 | 0.5255 | 1872.5 | -2.1678 | 0.5327 | 1906.5 | 3.2647 | 0.4283 | 1940.5 | 0.9642 | 0.3845 | 1974.5 | 2.6885 | 0.0821 |
| 1839.5 | 0.0568 | 0.5222 | 1873.5 | -1.6655 | 0.5205 | 1907.5 | 3.5239 | 0.4350 | 1941.5 | 1.1406 | 0.3885 | 1975.5 | 2.6103 | 0.0801 |
| 1840.5 | -0.0496 | 0.5222 | 1874.5 | -1.3436 | 0.5052 | 1908.5 | 3.6558 | 0.4375 | 1942.5 | 1.4046 | 0.3798 | 1976.5 | 2.7842 | 0.0816 |
| 1841.5 | -0.0939 | 0.5298 | 1875.5 | -1.4100 | 0.4963 | 1909.5 | 3.6327 | 0.4293 | 1943.5 | 1.5396 | 0.3666 | 1977.5 | 2.6479 | 0.0781 |
| 1842.5 | -0.0441 | 0.5401 | 1876.5 | -1.5422 | 0.4911 | 1910.5 | 3.6758 | 0.4242 | 1944.5 | 1.4817 | 0.3606 | 1978.5 | 2.7312 | 0.0810 |
| 1843.5 | 0.2966 | 0.5493 | 1877.5 | -1.5307 | 0.4852 | 1911.5 | 3.6258 | 0.4230 | 1945.5 | 1.4117 | 0.3585 | 1979.5 | 2.4677 | 0.0778 |
| 1844.5 | 0.5219 | 0.5447 | 1878.5 | -1.1921 | 0.4839 | 1912.5 | 3.8311 | 0.4235 | 1946.5 | 1.2692 | 0.3580 | 1980.5 | 2.1881 | 0.0710 |
| 1845.5 | 0.5765 | 0.5303 | 1879.5 | -0.8910 | 0.4825 | 1913.5 | 3.6079 | 0.4221 | 1947.5 | 1.1571 | 0.3591 | 1981.5 | 2.0775 | 0.0662 |
| 1846.5 | 0.4953 | 0.5121 | 1880.5 | -0.3036 | 0.4723 | 1914.5 | 3.2419 | 0.4244 | 1948.5 | 1.0998 | 0.3657 | 1982.5 | 2.1288 | 0.0565 |
| 1847.5 | 0.5554 | 0.5002 | 1881.5 | -0.1007 | 0.4687 | 1915.5 | 2.7533 | 0.4296 | 1949.5 | 1.0792 | 0.3659 | 1983.5 | 2.2874 | 0.0513 |
| 1848.5 | 0.5234 | 0.4900 | 1882.5 | -0.1856 | 0.4654 | 1916.5 | 2.8226 | 0.4337 | 1950.5 | 1.2933 | 0.3620 | 1984.5 | 1.5807 | 0.0469 |
| 1849.5 | 0.3381 | 0.5003 | 1883.5 | -0.2439 | 0.4618 | 1917.5 | 2.6845 | 0.4383 | 1951.5 | 1.2414 | 0.3570 | 1985.5 | 1.5632 | 0.0394 |
| 1850.5 | 0.2487 | 0.5125 | 1884.5 | -0.2376 | 0.4567 | 1918.5 | 1.9193 | 0.4519 | 1952.5 | 1.2913 | 0.3511 | 1986.5 | 1.3624 | 0.0345 |
| 1851.5 | 0.5245 | 0.5300 | 1885.5 | -0.2336 | 0.4589 | 1919.5 | 1.2874 | 0.4674 | 1953.5 | 0.9279 | 0.3471 | 1987.5 | 1.5002 | 0.0339 |
| 1852.5 | 0.7314 | 0.5399 | 1886.5 | -0.1307 | 0.4672 | 1920.5 | 1.2601 | 0.4722 | 1954.5 | 0.9608 | 0.3563 | 1988.5 | 1.4638 | 0.0325 |
| 1853.5 | 0.5653 | 0.5493 | 1887.5 | -0.1788 | 0.4880 | 1921.5 | 1.4201 | 0.4584 | 1955.5 | 0.8295 | 0.3014 | 1989.5 | 1.6527 | 0.0313 |
| 1854.5 | 0.4760 | 0.5503 | 1888.5 | -0.4181 | 0.4962 | 1922.5 | 1.0071 | 0.4374 | 1956.5 | 0.8690 | 0.1295 | 1990.5 | 2.0372 | 0.0299 |
| 1855.5 | 0.4888 | 0.5561 | 1889.5 | -0.5222 | 0.4992 | 1923.5 | 0.6224 | 0.4121 | 1957.5 | 1.2435 | 0.1207 | 1991.5 | 2.0828 | 0.0292 |
| 1856.5 | 0.3615 | 0.5556 | 1890.5 | -0.3298 | 0.4860 | 1924.5 | 0.4644 | 0.3950 | 1958.5 | 1.2559 | 0.1201 | 1992.5 | 2.2246 | 0.0271 |
| 1857.5 | 0.0566 | 0.5546 | 1891.5 | -0.1843 | 0.4781 | 1925.5 | 0.4575 | 0.3865 | 1959.5 | 1.1627 | 0.1190 | 1993.5 | 2.3237 | 0.0279 |
| 1858.5 | -0.1613 | 0.5505 | 1892.5 | -0.1106 | 0.4622 | 1926.5 | 0.3247 | 0.3786 | 1960.5 | 1.0440 | 0.1182 | 1994.5 | 2.0950 | 0.0288 |
| 1859.5 | -0.2636 | 0.5409 | 1893.5 | -0.0512 | 0.4395 | 1927.5 | 0.0435 | 0.3689 | 1961.5 | 0.9789 | 0.1169 | 1995.5 | 2.1790 | 0.0270 |
| 1860.5 | -0.4459 | 0.5263 | 1894.5 | 0.3489 | 0.4104 | 1928.5 | -0.1716 | 0.3666 | 1962.5 | 1.2095 | 0.1093 | 1996.5 | 1.6845 | 0.0280 |
| 1861.5 | -0.7894 | 0.5119 | 1895.5 | 1.0009 | 0.3956 | 1929.5 | -0.0981 | 0.3691 | 1963.5 | 1.4774 | 0.1095 | 1997.5 | 1.6924 | 0.0279 |
| 1862.5 | -0.9662 | 0.4933 | 1896.5 | 1.5798 | 0.4020 | 1930.5 | -0.0457 | 0.3700 | 1964.5 | 1.9331 | 0.1066 | | | |
| 1863.5 | -1.2553 | 0.4812 | 1897.5 | 1.9823 | 0.4070 | 1931.5 | 0.1467 | 0.3656 | 1965.5 | 2.2509 | 0.1058 | | | |
| 1864.5 | -1.7841 | 0.4763 | 1898.5 | 2.4358 | 0.4058 | 1932.5 | 0.1193 | 0.3587 | 1966.5 | 2.5171 | 0.1062 | | | |
| 1865.5 | -2.2262 | 0.4728 | 1899.5 | 2.7879 | 0.4008 | 1933.5 | 0.1679 | 0.3499 | 1967.5 | 2.4852 | 0.0998 | | | |

**Figure 1.** Plots of the LUNAR97 length-of-day values (a) and of the uncertainty in their determination (b). Since LUNAR97 is formed by concatenating the smoothed lunar occultation, Hipparcos optical astrometric, and COMB97 length-of-day series, the LUNAR97 values and uncertainties during 1832.5–1955.5 are simply those of the smoothed lunar occultation LOD series, during 1956.5–1961.5 they are those of the smoothed Hipparcos optical astrometric LOD series, and during 1962.5–1997.5 they are those of the smoothed COMB97 LOD series.

**Figure 2.** Plots of the LUNAR97 excess length-of-day values and $1\sigma$ standard errors (black curve with gray shading representing $\pm 1\sigma$ standard error), the excess LOD series of McCarthy and Babcock (1986; red curve), and the excess LOD series of Stephenson and Morrison (1984; green curve). Note that after about 1984 the uncertainties of the LUNAR97 LOD values are less than the width of the black line. The differences between these 3 series, which occur primarily at the higher frequencies, can be attributed to differences in the lunar ephemeris, reference frame, analysis technique, and degree of smoothing used when reducing the lunar occultation, optical astrometric, and space-geodetic measurements for excess length-of-day.

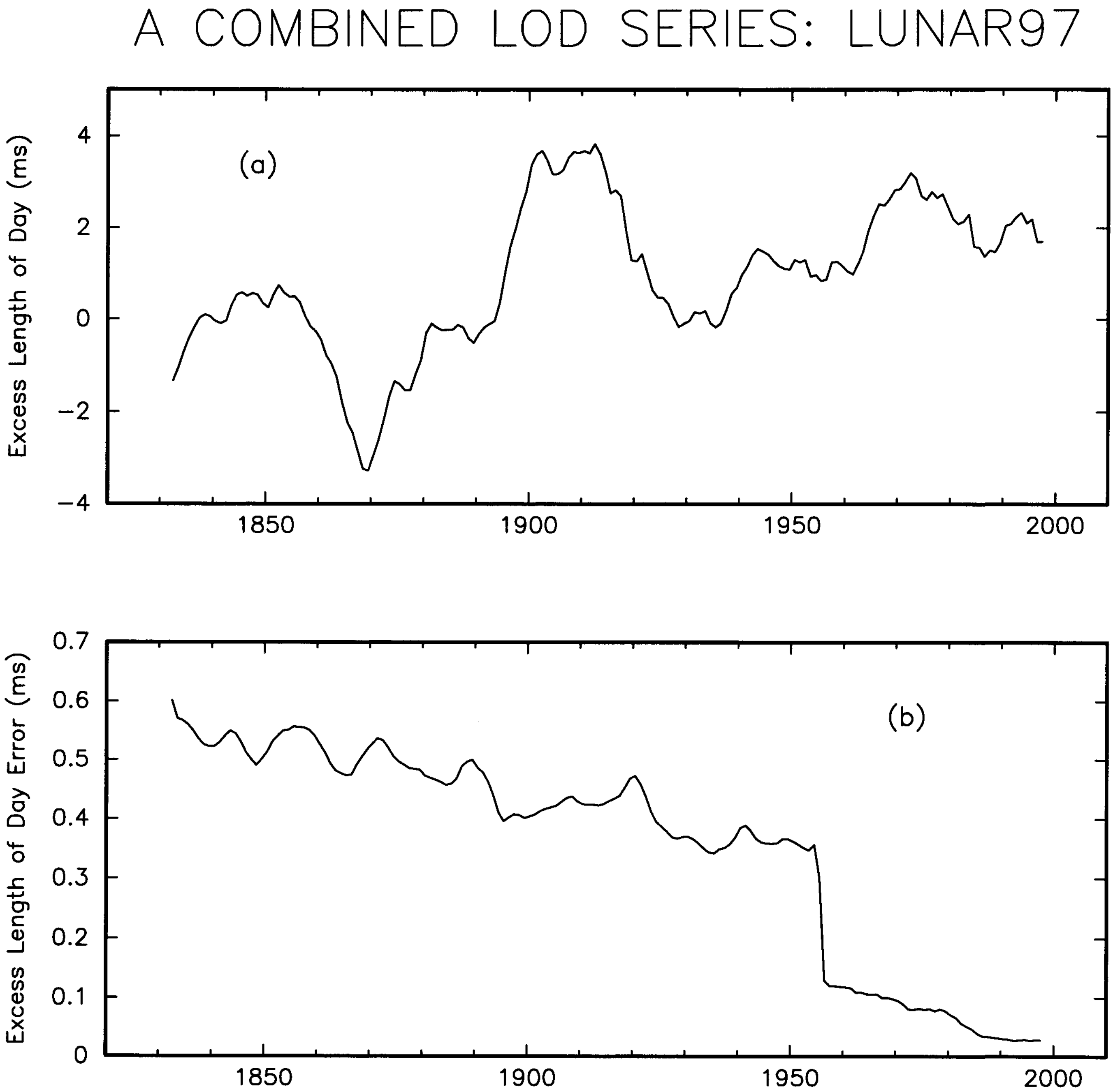
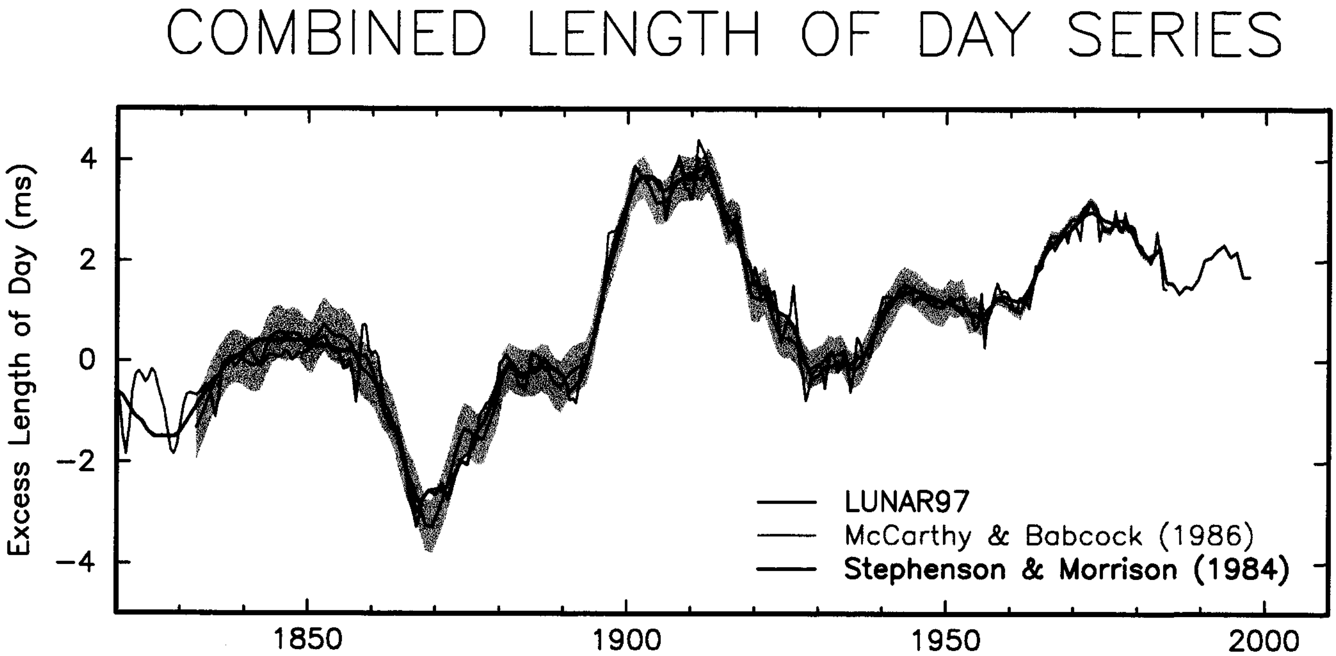

A COMBINED LOD SERIES: LUNAR97
(a)
Excess Length of Day (ms)
4
2
0
−2
−4
1850
1900
1950
2000
(b)
Excess Length of Day Error (ms)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1850
1900
1950
2000

Figure 1



COMBINED LENGTH OF DAY SERIES
Excess Length of Day (ms)
4
2
0
−2
−4
1850
1900
1950
2000
LUNAR97
McCarthy & Babcock (1986)
Stephenson & Morrison (1984)


Figure 2